

Phase transition in the SRG flow of nuclear interactions ^{*}

V. S. Timóteo · E. Ruiz Arriola · S. Szpigel

Received: date / Accepted: date

Abstract We use a chiral interaction at N³LO in the 1S_0 channel of the nucleon-nucleon interaction in order to investigate the on-shell transition along the similarity renormalization group flow towards the infrared limit. We find a crossover at a scale that depends on the number of grid points used to discretise the momentum space.

Keywords Phase Transition · Similarity Renormalization Group · Nuclear Force

One of the most appealing features of nature is universality. Some phenomena disguise themselves across many different areas where physical systems are described by sometimes unrelated theories or models. Yet they appear recurrently in some form.

An excellent example is the phase transition resulting from a broken symmetry. It is observed in magnetism when the temperature of a spin chain in a two-dimensional Ising model crosses a critical value [1]. It appears in nuclear physics when observing rotational spectra of deformed nuclei [2] and it is also present in hadron physics when the coupling between quarks in a two-flavour NJL model exceeds a critical value [3].

In both magnetism and nuclear physics the phase transition results from the breaking of the rotational symmetry and the corresponding Goldstone bosons are spin waves and nuclear rotation. In hadron physics the phase transition results from the chiral symmetry breaking and the corresponding Goldstone boson is the

^{*} Presented by VST at the 23rd European Conference on Few-Body Problems in Physics, Aarhus, Denmark, 08 - 12 August 2016.

V. S. Timóteo
Faculdade de Tecnologia, Universidade Estadual de Campinas - UNICAMP
13484-332, Limeira, São Paulo, Brasil

E. Ruiz Arriola
Departamento de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada
E-18071 Granada, Spain

S. Szpigel
Centro de Rádio-Astronomia e Astrofísica, Universidade Presbiteriana Mackenzie
01302-907, São Paulo, SP, Brasil

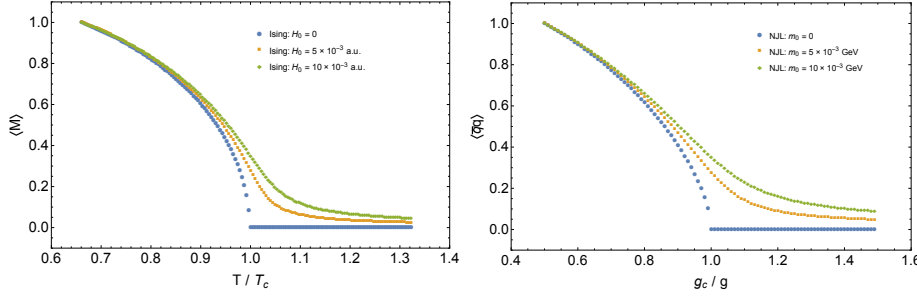


Fig. 1 Universal phase transitions: the Ising model (left) and the NJL model (right). The critical value separates the Wigner-Weyl and Nambu-Goldstone phases. Order parameters are normalized so that they start from one.

pion. This phenomenon is shown in Fig. 3 for both two-dimensional Ising model (left) and two-flavour NJL model (right).

In this work we report on a similar and remarkable phase transition observed in the similarity renormalization group flow, which is used to change and calibrate the resolution scale of nuclear interactions to their natural values in different applications.

The evolution of an NN interaction with the SRG [4] is performed by numerically integrating the Wegner renormalization group flow equation for the potential matrix

$$\begin{aligned} \frac{d V_s(p, p')}{ds} = & -(p^2 - p'^2)^2 V_s(p, p') + \frac{2}{\pi} \int_0^\infty dq q^2 \\ & \times (p^2 + p'^2 - 2q^2) V_s(p, q) V_s(q, p') \end{aligned} \quad (1)$$

where $s = 1/\lambda^4$ and λ is the similarity cutoff. The flow equation generates a set of isospectral interactions that approaches a diagonal form as $s \rightarrow \infty$ (or $\lambda \rightarrow 0$). Here we explore the use of the Frobenius Norm (FN) of the potential as a measure of the on-shellness of the interaction in order to study the on-shell transition as the interaction undergoes the renormalization group flow through Wegners equation. For a real self-adjoint potential, $V_\lambda(p', p) = V_\lambda(p, p')$ the FN, ϕ_λ , is defined by means of the formula

$$\phi_\lambda^2 = \|V_\lambda\|^2 = \left(\frac{2}{\pi}\right)^2 \int_0^\infty dq q^2 \int_0^\infty dp p^2 [V_\lambda(p, q)]^2. \quad (2)$$

As an order parameter for the on-shell transition, we consider the derivative of the Frobenius norm with respect to the similarity cutoff

$$\beta = \frac{\partial \phi}{\partial \lambda}, \quad (3)$$

and to find the transition scale we look at the derivative of the order parameter

$$\eta = \frac{\partial \beta}{\partial \lambda}. \quad (4)$$

This procedure is same as the one used to study, e.g., the chiral (crossover) transition of QCD, where the order parameter is the chiral condensate, the running

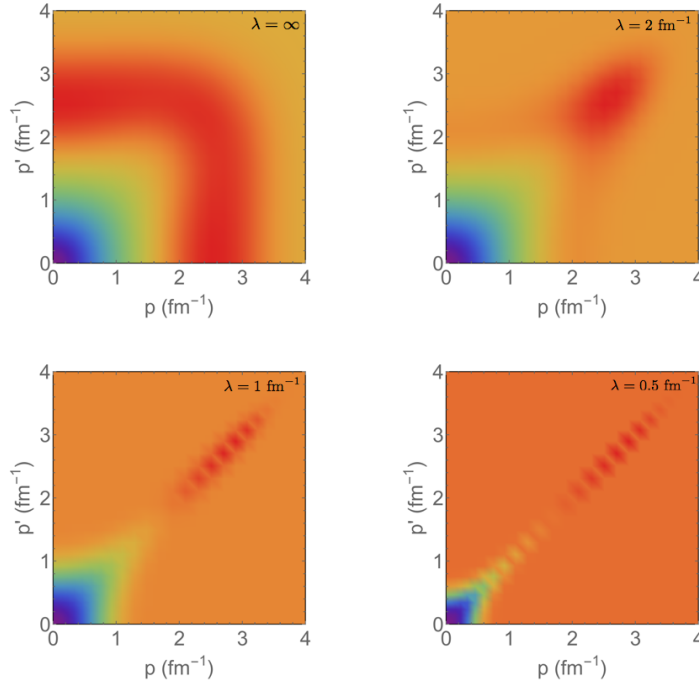


Fig. 2 SRG evolution of the N3LO chiral potential in the 1S_0 channel towards the infrared region of the similarity cutoff.

scale is the temperature and the transition temperature is obtained from the thermal susceptibility, which is the derivative of the condensates with respect to the temperature [5].

Since we are interested in the infrared region of the similarity cutoff, we choose the N3LO chiral potential of Entem and Machleidt [6] due to its short tail in momentum space which makes the SRG evolution for $\lambda \rightarrow 0$ more practical. In the case of the N3LO potential, we can work with a maximum momentum of 4 fm^{-1} and $N = 10, 20, 30$ points for discretisation. This setup gives very reasonable values of the two-body contribution for the Triton and Helium binding energies as we approach typical values for the similarity cutoff. Following our previous works [7, 8, 9, 10], where a toy model for the nuclear force was used, we perform here the evolution of the N3LO potential in the range $0 < \lambda < 2 \text{ fm}^{-1}$, generating a family of phase-equivalent effective potentials $V_\lambda(p, p')$ partially shown in Fig. 2.

In the upper left panel of Fig. 3 we show the running of the Frobenius norm with the similarity cutoff for different numbers of grid points. The norm drops very rapidly till it becomes stationary, indicating the onset of the on-shell limit. The order parameter β is displayed in the upper right panel of Fig. 3, where we observe a clear crossover. We can locate the transition scale by looking at the derivative of the order parameter. We call this quantity similarity susceptibility and denote it by η_λ , in analogy to the thermal susceptibility χ_T of QCD. The similarity susceptibility is shown in the lower left panel of Fig. 3, revealing the expected behaviour for a crossover transition: a well-defined peak of the susceptibility at

a similarity cutoff λ_c . As the number of grid points increases, the peaks of the similarity susceptibility become narrower, higher and at lower λ_c . For $N = 30$ grid points, the peak of η_λ is at $\lambda_c \sim 0.9 \text{ fm}^{-1}$. The critical similarity susceptibility for different number of grid points is displayed in the lower right panel of Fig. 3.

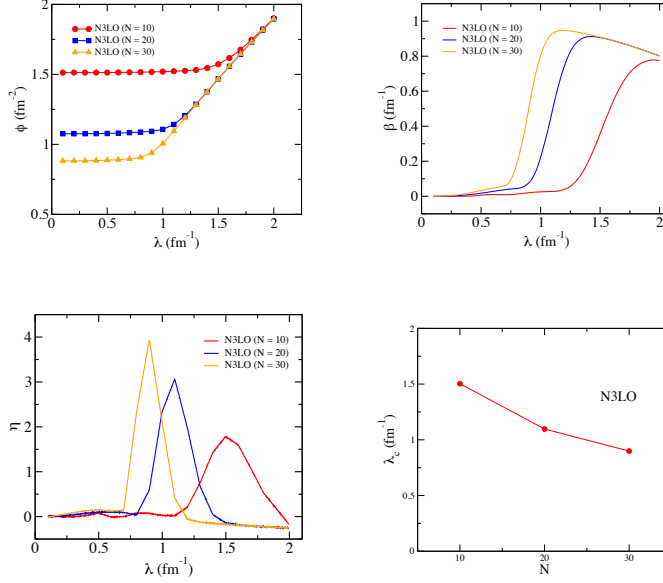


Fig. 3 Frobenius norm ϕ (upper left), order parameter $\beta = \frac{\partial \phi}{\partial \lambda}$ (upper right) and similarity susceptibility $\eta = \frac{\partial \beta}{\partial \lambda}$ (lower left) as functions of the similarity cutoff and the critical similarity cutoff λ_c as a function of the number of grid points N (lower right).

Summarising, we have studied the on-shell transition of the similarity renormalization group flow for a N3LO chiral potential. Our results indicate a crossover at a critical similarity cutoff λ_c approximately 0.9 fm^{-1} , below which we can consider the interaction to be on-shell. The usefulness of this observation is that in a system with a finite size, such as the atomic nucleus, where the momentum is naturally discretized, the on-shell limit is reached rather quickly. The finite momentum resolution Δp imposes an infrared cut-off which effectively builds the on-shell limit for $\lambda \gg \Delta p$.

Acknowledgements

We would like to thank Spanish Mineco (FIS2014-59386-P), Junta de Andalucia (FQM225), FAPESP (2016/07061-3 & 2016/05554-2) and CNPq (306195/2015-1).

References

1. L. Onsager, Phys. Rev. 65 (1944) 117.

2. S. Frauendorf, *Rev. Mod. Phys.* 73 (2001) 463.
3. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* 122 (1961) 345.
4. S. K. Bogner, R. J. Furnstahl, A. Schwenk, *Prog. Part. Nucl. Phys.* 65 (2010) 94.
5. R. Farias, V. Timteo, S. Avancini, M. Pinto and G. Krein, [arXiv:1603.03847](https://arxiv.org/abs/1603.03847).
6. R. Machleidt and D. R. Entem, *Phys. Rept.* 503 (2011) 1.
7. E. Ruiz Arriola, S. Szpigel and V. Timteo, *Phys. Lett. B* 728 (2014) 596.
8. E. Ruiz Arriola, S. Szpigel and V. Timteo, *Phys. Lett. B* 735 (2014) 149.
9. E. Ruiz Arriola, S. Szpigel and V. Timteo, *Annals of Physics* 353 (2015) 129.
10. E. Ruiz Arriola, S. Szpigel and V. Timteo, *Annals of Physics* 371 (2016) 398.